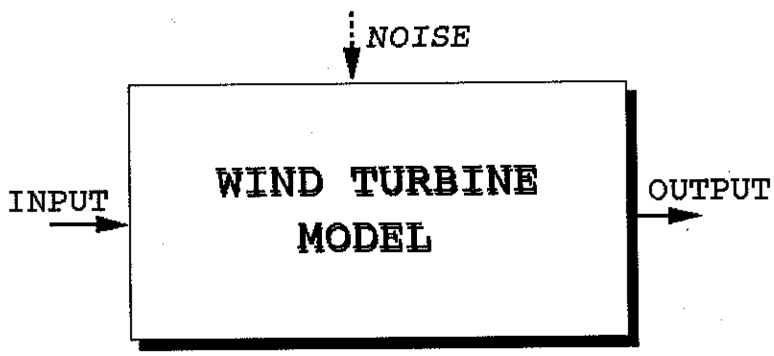
WIND TURBINES MODELLING:
An introduction to experimental identification

Cristian Tantareanu *



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Preface

This report presents the general and basic problems of the experimental identification with application on wind turbines operation process.

It synthesizes a part of the work in this topic carried out by the author in order to prepare his PhD with the Technical University Timisoara TUT.

At the same time the report aims to signal the opportunity of the topic to be considered at present and future Folkecenter projects. The experimental research on the Folkecenter 500 kW wind turbine opens a good application field.

The author thanks to Folkecenter for Renewable Energy for the offered Visiting Researcher stage and for the excellent conditions which made possible, beside the current activity with the Folkecenter projects, this additional work.

Folkecenter for Renewable Energy
Ydby, Denmark

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C O N T E N T S

Page no.

1. INTRODUCTION

Modelling the behaviour of medium and large wind turbine (WT) is essential for optimum design and operation control. It is specific for WT to work in a continuous strong transitory regime due to the rapid and stochastic variation of the wind speed. For that reason the model of a WT process should be particularly detailed.

More than that, in the model of the process should be included the wind model too as the wind acts like a stochastic perturbation.

The paper presents in a condensed way an approach on the experimental identification of WT models: the purposes, the modern adaptive control concept which assumes on line modelling, the basic mathematical technics of the experimental identification, how is the wind included in the modelling.

Wide attention is paid now on the experimental identification topic in very different fields. A variety of methods are developed for the mathematical techniques, especially for the recursive identification . In the annex a basic approach is made.

Some results known in the literature for both analytical and experimental models for medium and large WT are given in the chapter 3. Here are also presented the author's softwares for WT experimental identification and applications on simulated input data.

The paper is meant as an introduction in the problem and tries, after giving basic guidance , to identify how a further experimental identification work could be continued and applied with the Folkecenter activity on the 5oo kW WT pilot.

2. MODELLING A WIND TURBINE . WAYS AND PURPOSES.

The rapid variability of the wind parameters implies a high WT model in order to make it capable to give the appropriate answer. Another particular aspect that rises the complexity of the model is the constructive elasticity of modern designed WT, it means more freedom degrees.

The WT modelling is required to simulate specific regimes and thus avoiding real tests, expensive, long lasting and difficult to meet. The model helps in optimizing the design of the WT and of the control equipment. In the operation control process , the WT model is used to predict the regime in short term and thus to optimize it; one follows criteria as reducing mechanical or thermal stresses, maximum energy output, limiting the output power, minimum cut-in/cut-out maneuvers a.o.

To obtain a WT model two ways are possible:

-analytical identification
-experimental identification

The analytical identification starts from the physical laws of the process and build the mathematical functions which link the input with the output. The analytical identification should be made for every component equipment of a WT. After that one links that partial models in a global one.
The WT analytical identification could be extremely difficult especially to identify the behaviour of the aerodynamic part , the rotor. The complexity of the global model is high, making it improper to be used in real time unless important simplifications are made.

Even so, the most difficult part is still to come: the analytical modelling of the real spatial wind speed, all over the rotor plane . One starts from a punctual known value as measured by an anemometer, sited usually at some distance from the WT, in specific terrain conditions.

The experimental identification is a technique trying to overcome the analytical difficulties by approaching the problem directly and in a global manner. For example if we are interested to catch the wind speed - power relation, one starts from a real experimental set of data as measured simultaneously from the anemometer and from the WT output. By appropriated mathematical techniques one identifies the transfer function wind speed-power which gives the best approach to the real data. Modern mathematical techniques relay on statistical methods in order to eliminate the noise influence in the process and in the measurements.
In this concept the WT process is regarded as a black box with input (in our example) the wind speed measured punctual by the anemometer and with output the power. The black box includes, beside the WT model, the model of the spatial change of the wind speed between the value measured by the transducer and an

equivalent constant value in the WT rotor plane.

Therefore the model of the WT is seen global and pragmatic, including the placement of the anemometer and the site wind flow specific.

The same approach could be made for other single input-single output (SISO) WT models as:

wind speed-mechanical torque
wind speed-generator winding temperature
rotor speed-power
yaw angle-power

The models identification could be obtained in real time too and used for control purposes, predicting the controlled values.The model obtained on line by experimental identification could be corrected permanently taking into account the last real measurements compared with the predicted ones. It means a self tuning (or adaptive) control. The general scheme for such a control is given in fig.1.

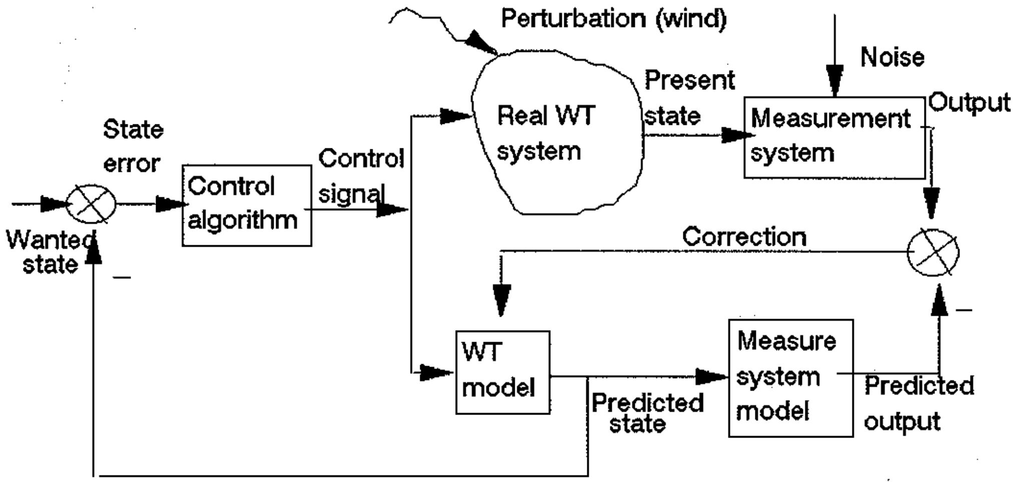

Fig.1. State control diagram.

This modern adaptive control is implemented now on some British and German WT units to improve the pitch regulation /1,2,3,4, 31/.
For stall regulated machines this control could be addressed for
other purposes like yaw orientation or variable speed control.

Another application is to predict the wind speed values in order
to control the cut-in/cut-out maneuvers. The classical concept that the control will react according to controlled values known from already past periods (the persistence concept) leads generally to some unnecessary additional maneuvers /5/.

3. APPLICATIONS

3.1. Wind prediction

Several applications of wind prediction methods on real data are known /12,13,14 /.

The general conclusions were:

-the error predictions depend on the specific site,
-the error predictions are not sensible with the average wind speed value,
-the optimum order of the Kalman filter is 4 or 5. For orders greater then 6 the improvements are not important despite the calculation efforts
-the minimal prediction errors appear for sets of data averaged on 1 to 5 minutes intervals
-if the data are averaged on more than 1 hour interval the prediction by Kalman or the auto regressive models is not efficient. For this prediction intervals, the persistence model is sufficient.
-optimal parameters for the ARMA model are (1,5): regression order 1, moving average order 5.

Wind prediction practical results on the optimization of the cut-in maneuvers strategy for a large WT are shown in the figures no. 2 and no 3.

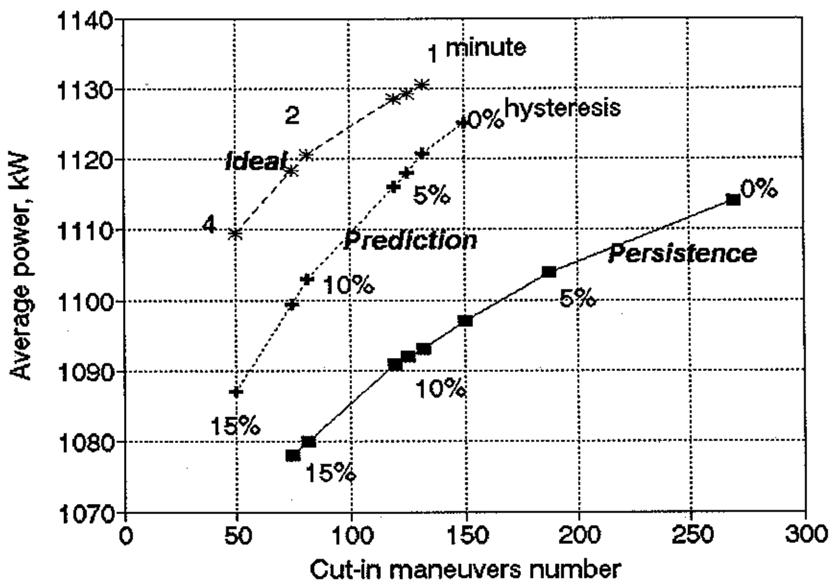

Fig.2. Improvement of the cut-in regime using a prediction strategy

The figure no.2 shows the results for a control strategy combining an ARMA model prediction with hysteresis. The optimum compromise between the energy output and the maneuvers number is reached for a hysteresis of 6..8%.

From the figure 3 one sees that the Kalman prediction control increases the output energy simultaneous with less starting maneuvers. The optimum seems to be obtained for a Kalman prediction combined with a hysteresis value of 7-8%.

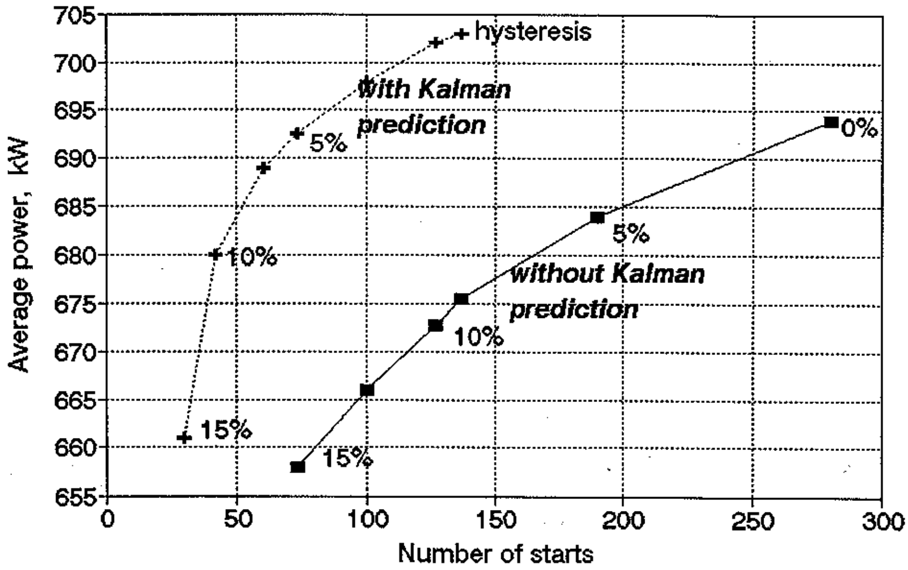

Fig.3 Start maneuvers during a 720 hours operation period

3.2. Windturbines models

3.2.1. Analytical models

An advanced model called DUWECS was developed at the Technical University Delft, Holland /15,16,17,18,19/. One takes into consideration a flexible, 2 bladed, upwind, variable speed WT. The resulting equations system is very complex and can not be used for on line control purposes.

At Strathclyde University more simplified models were adopted as shown on the figures 4 and 5 /20, 21/.

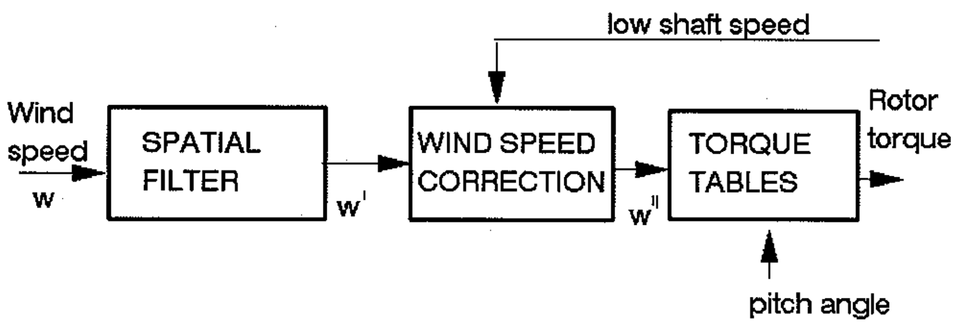

Fig.4. A model for the aerodynamic part /20/

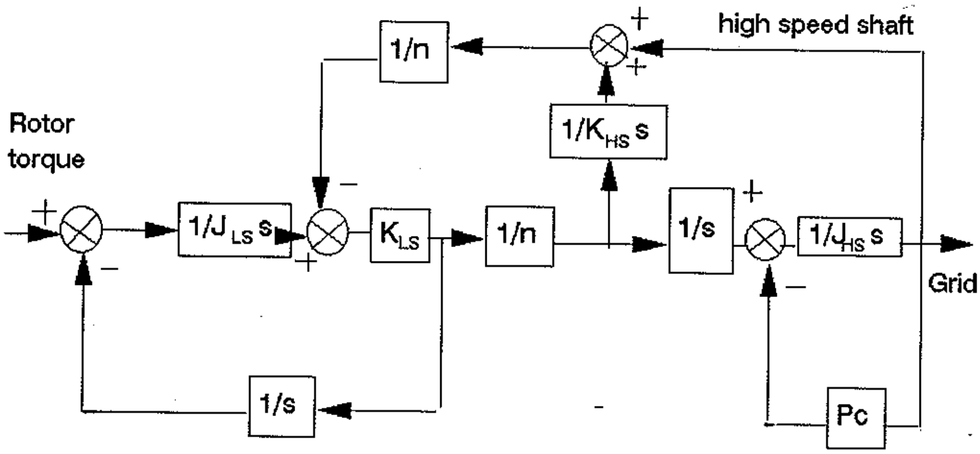

Fig.5. The drive train model /20/

J is the inertia, K the stiffness, and Pc the slope of the electrical torque curve.

In this model the punctual measured value of the wind speed is processed through a spatial filter in order to obtain an equivalent wind speed value as seen by the rotor. The transfer function of this filter is:

$$G(s) = \frac{\sqrt{2} + bs}{(\sqrt{2} + b\, s\, \sqrt{0.55})(1 + s\, b/\sqrt{0,55})}$$

with $b = \tau R/V$
$\tau = 1.2$ wind speed variation factor with the rotor height

The transfer function between the load torque and the rotor torque is:

$$\frac{Mg}{Mrot} = \frac{b1*s + bo}{a3*s^3 + a2*s^2 + a1*s + ao}$$

where

$b0 = K_{LS}K_{HS}n^2\,Pc$ $\qquad$ $b1 = K_{LS}K_{HS}n^2 J_{HS}$

$ao = n^2\,K_{LS}K_{HS}Pc$

$a1 = K_{LS}K_{HS}(J_{LS} + n^2\,J_{HS})$ $\qquad$ $a2 = J_{LS}\,Pc\,(k_{LS} + n^2\,K_{HS})$

$a2 = J_{LS}J_{HS}(K_{LS} + n^2 K_{HS})$

Further developments /22/ conducted to a 3oo kW WT model as presented in the figure 6:

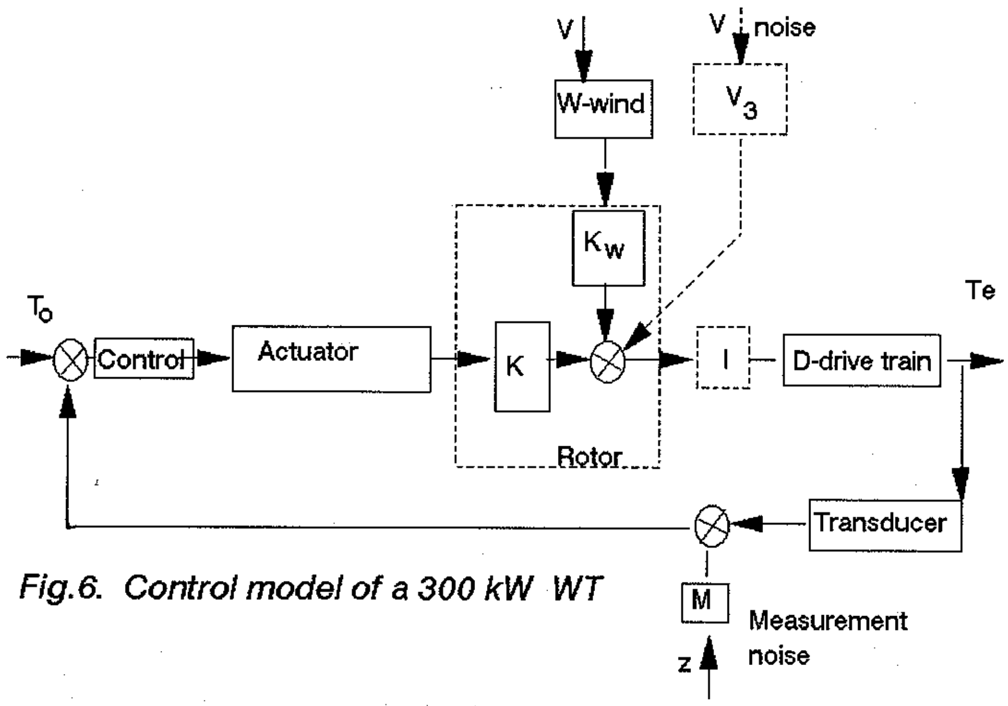

Fig.6. Control model of a 300 kW WT

The v and z inputs are white noise.

The transfer functions are:

-actuator $A = 25.9/(s+25.9)$

-controller $$C = \frac{17(s^2 + 2s + 2,107}{s(s+0,163)(s+3,246)^2}$$

-wind model $$W = \frac{1,3\ (1+0,86s)}{(s+0,086)(1+0,64s)(1+1,64s)}$$

-drive train+generator $$D = \frac{2123,38}{s^4 + 33,39s^3 + 7566,13s^2 + 6421,3s + 8090}$$

- torque transducer $T = 1/(1+0,02s)$

The aerodynamic gains K and K_W are 8090 and 16 840.
The measured noise intensity M=1.

This model was completed /1/ such that:

-to the wind model was added the aerodynamic perturbation given by blades in relation with the tower:

$V3 = 7350(s+12.66) / (s^2 + 3s + 160,2)$

- the induction lag effect in the rotor wake given by the wind speed change, the rotor speed change or the pitch angle change was modelled as:

$$I(s) = (11{,}25s + 1)/(7{,}5s + 1)$$

A more simplified model /3/ aiming to a control in state space is:

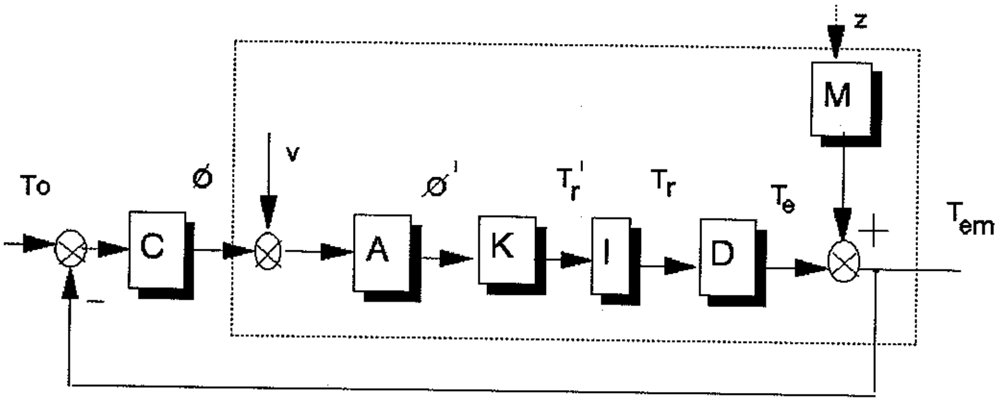

Fig.7. A simplified WT control model /3/.

where

actuator

$$A = \frac{1{,}029 \cdot 10^{5} s + 2{,}057 \cdot 10^{8}}{s^{4} + 4{,}85 \cdot 10^{2} s^{3} + 1{,}095 \cdot 10^{5} s^{2} + 1{,}047 \cdot 10^{2} s + 2{,}057 \cdot 10^{8}}$$

induction lag $I = (1+11{,}25s)/(1+7{,}5s)$

measurement noise intensity $M = 1$

gain $K = 8090$

drive train+generator $D = \dfrac{0{,}282}{s^{2} + 0{,}8064\, s + 10{,}745}$

Both stochastic inputs are white noise.

Another approach to model a WT in the state space is made by E.A. Bossany /23/.

3.2.2. Experimental identification

There are few references regarding experimental identification work on WT.

On the MODOA WT- 200 kW, an experimental identification was carried out based on correlation techniques /27/.
In the last two years experimental identification researches are done on the UNIVEX WT in Germany. UNIVEX has a 16 diameter

rotor and could operate in various regimes : fixed or variable speed, teetered or rigid hub, blades with rigid joins or with flexible ones, up-wind or down-wind rotor.
On this unit one verifies the analytical model DUWEC developed at Technical University Delft /28/.

The transfer function between the low speed shaft torque and the pitch angle was identified . The identified model has a Box-Jenkins structure and the optimum order is 5 for the determinist part and 7 for the stochastic (moving average) part.

3.3. Identification softwares

In this part one presents the application softwares developed by the author /30/.

3.3.1. The elaborated programmes

Some calculation programmes for experimental identification were developed by the author.

The programme CORRID permits a simple statistical processing of the power and wind speed data series. It gives the average, the

sample variance ,the auto-correlation ann cross-correlation coefficients and functions of two data series (wind speed and power). The programme solves the Wiener Hopf matricial equation for maximum 15 time values of the unit impulse response.

The programme LSQOFF uses the ordinary off-line least squares estimation method with variants of weighted LSQ and MARKOV weighted LSQ.Depending on the RAM computer capacity the programme permits the processing of series up to 1000 data.

The programme RECLSQ performs an ordinary recursive LSQ data processing. It includes also the weighting possibilities with the inverse of the regression equation covariance or with a forgetting factor.

Matricial operations facilities are provided by a standard TurboPascal mathematical library. The programmes use a graphic TurboPascal library.

3.3.2.Programmes applications on simulated data

The programmes were first tested in order to assure their reliability on simpler set data and comparative with test examples given in literature.

After that a study on simulated data was performed.
To simulate an artificial set of "wind speed - power" data, real wind speed data series collected at Rutherford Appleton Laboratory UK test site were used. The wind speed values were sampled at every 2 seconds. The correspondent power data series was simulated starting from the real wind speed data series using a simple linear autoregressive exogenous model of second order for the dependence between power and the wind speed:

$$p(t) = b2*w(t-2) + a2*p(t-2) + b1*w(t-1) + a1*p(t-1) + b0*w(t) + e(t)$$

were p, generated power,kW
w, wind speed,m/s
e , Gaussian error,kW

We considered here wind speed like an input.

The first 5 terms of the model form the deterministic part of the output power. This part we try to find through identification techniques from the perturbed power output. The noise "e" was simulated with :

$$e(t) = \sigma \cdot \sqrt{(-2 \ln y1) \cos (2 \pi y2)}$$

were y1 and y2 are independent random numbers from a uniform distribution on the interval (0,1). The noise series has the standard deviation σ and zero mean.

In this way , "wind speed-power" data series were created with different standard deviation noise included in the power value ; two sets of model parameters were considered:

	b2	a2	b1	a1	b0
autoregress.	1	0.0	-2	0.7	4
regressive	2	0.0	6	0.0	3

The first set is a strong autoregressive one with an important (a1) dependence from the previous output (power). The second set creates a regressive model, thus the power are to be dependent only from the wind speed. These two parameter sets were especially chosen to get evidence of the difficulties induced, in the first case, by the noise correlation with the measurements vector. The wind speed and the deterministic power output evolutions for the regressive and autoregressive model are shown in figures 8 and 9.

The standard deviation of the power noise is considered parametrical in the large range 1....40 kW. Figures 10 and 11 show examples of simulated perturbed power data with noise with standard deviation σ = 20 kW.

A graphic comparison between the nod-perturbed power data series, the noise perturbed power data series and the estimated

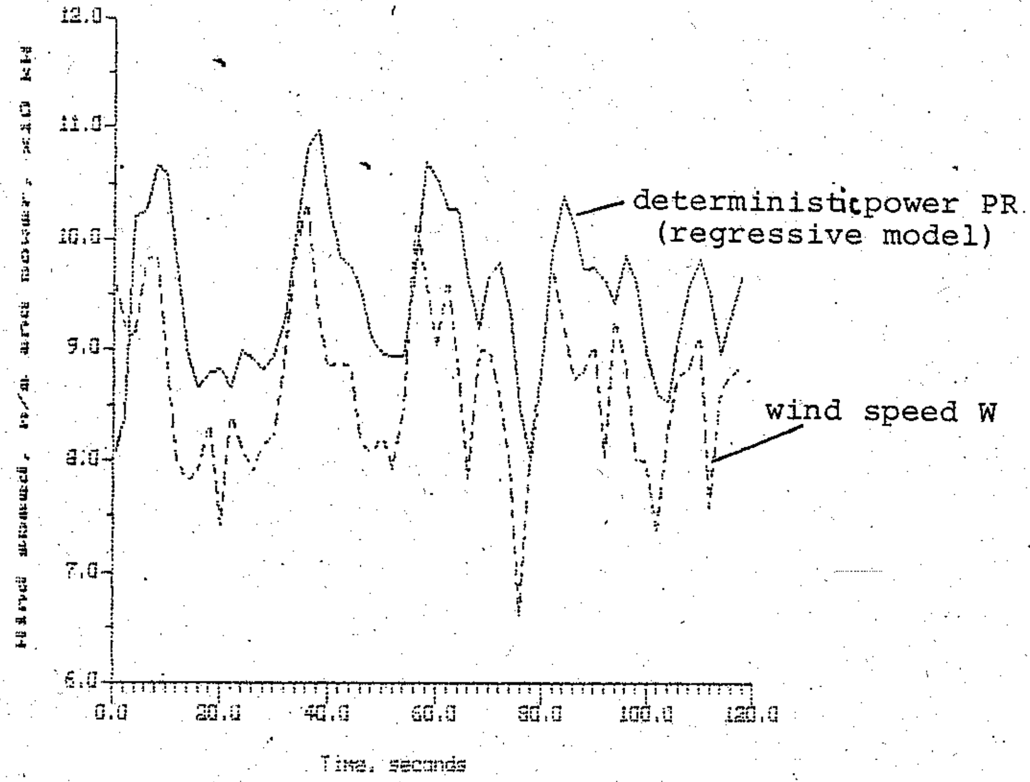

Fig.8. The wind speed and the deterministic power simulated by the regressive model

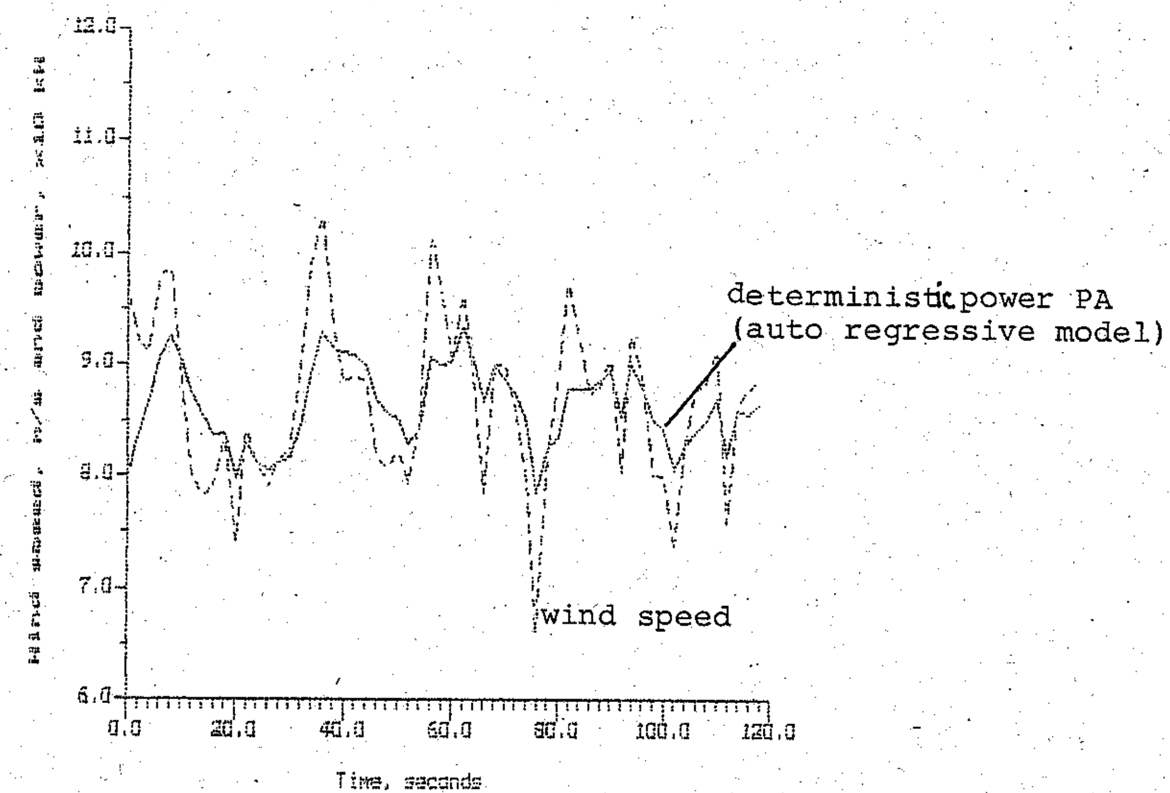

Fig.9. The wind speed and the deterministic power simulated by the autoregressive model

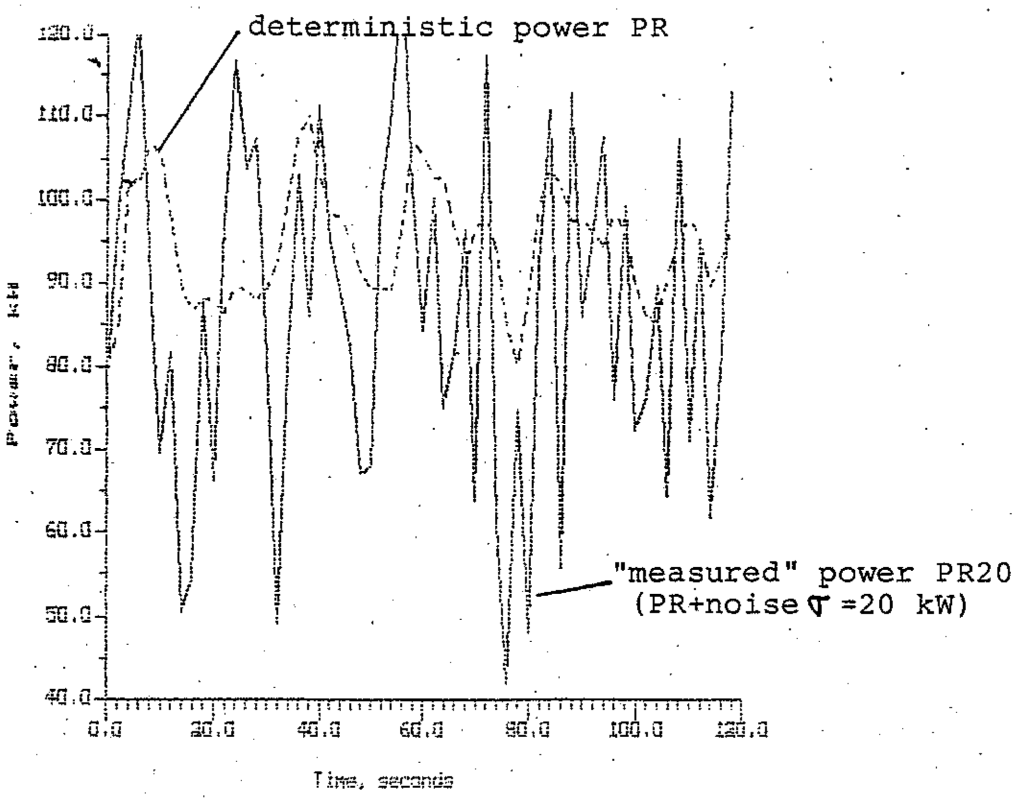

Fig.10. The deterministic power PR and the simulated "measured" power PR20 which includes a noise whith σ =20 kW

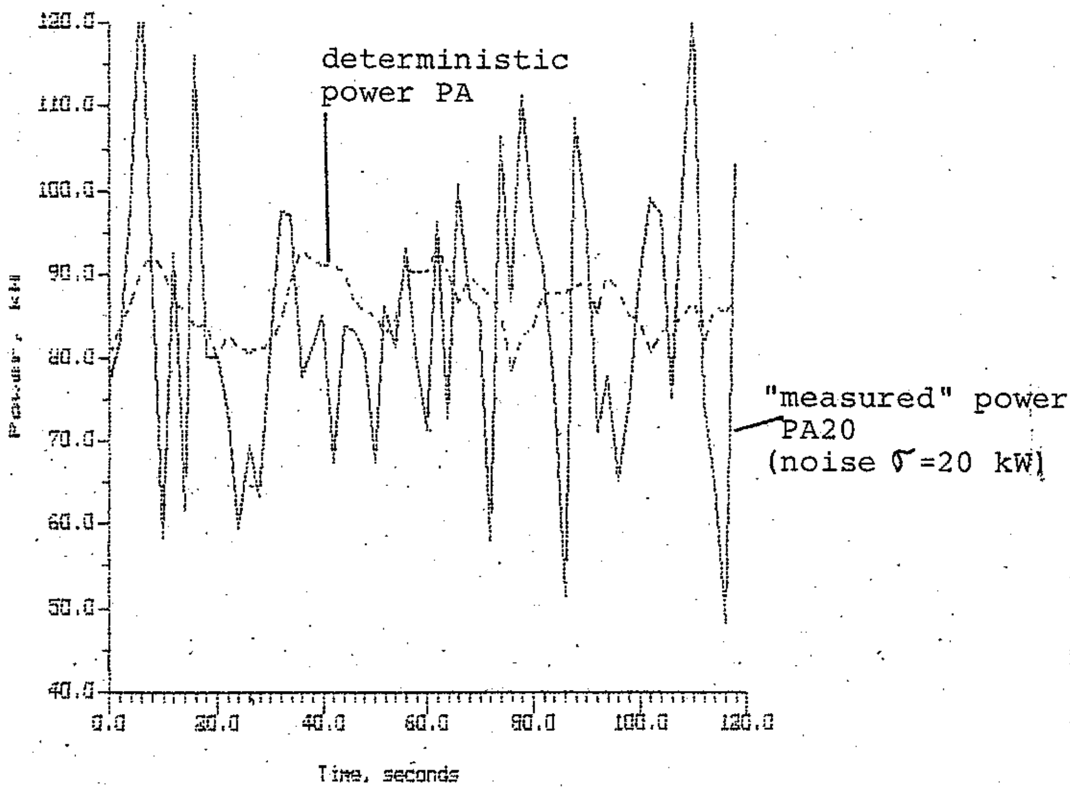

Fig.11. The deterministic power PA and the simulated "measured" power PA20 which includes noise whith σ =20 kW

power data series are shown following.

Figure 12 presents the estimated autoregressive power data obtained with the ordinary off-line least squares method (LSQOFF programme). The power data series predicted from the perturbed data are close to the non-perturbed data . Due to the correlation between the noise and the measurements vector, less acceptable deviations will appear at more increased noise perturbations then the actual considered $\sigma = 20$ kW.

In figure 13 one can see that for the regressive model it is possible to extract the non-perturbed data series from much higher perturbations.

More important for practical control applications is the recursive algorithm, which rises also the problem of convergence.

The recursive estimated autoregressive power , after approximately 110 steps (220 sec), is acceptable close to the real power (figure 14).

The regressive model reacts better (in approximately 90 steps) then the recursive algorithm as could be seen in figure 15.

Regarding the parameters convergence , the autoregressive model parameters could not converge to their real value due to the mentioned correlation of the noise with the measurement vector. The Gaussian noise imposes an autoregressive coefficient a1 near to zero. Only the b0 wind speed coefficient converges to the real value (b0=4) (figure 16). It is obviously necessary to use in this case more elaborated estimation techniques then the ordinary LSQ, as the variable instrumentals algorithm etc.

For the regressive model (figure 17) the parameters do converge to their real values but extremely slow. Here more elaborated algorithms to improve convergence should be employed too.

The conclusions of this first tests are that the basic ordinary off line and recursive LSQ methods give promising results in wind turbines power data estimation but further refinements should be developed in order to assure and improve the algorithm convergence.

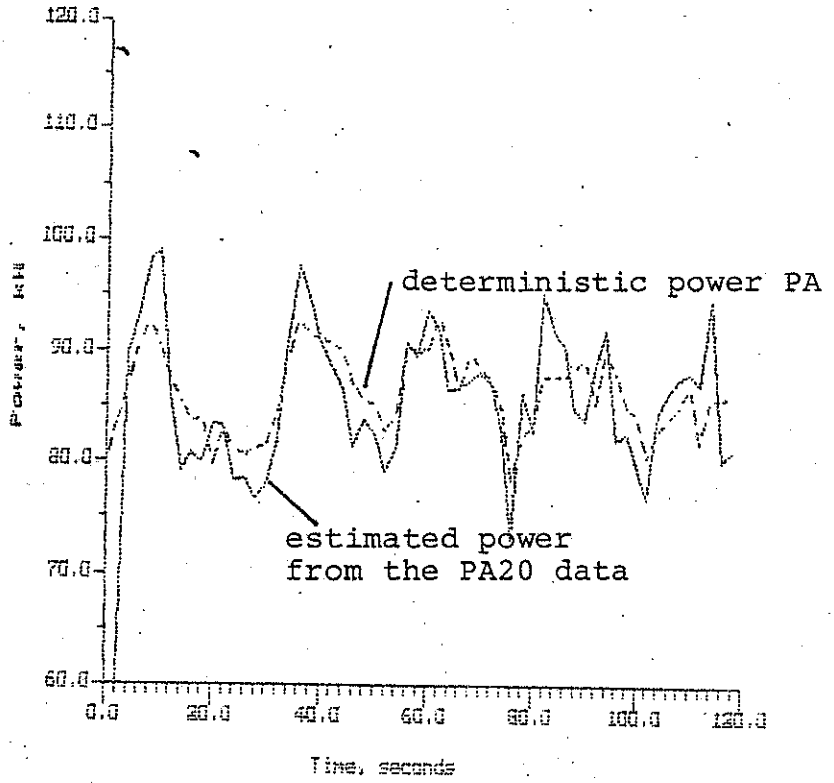

Fig.12. Estimated power PA versus real power. Batch (off-line) algorithm.

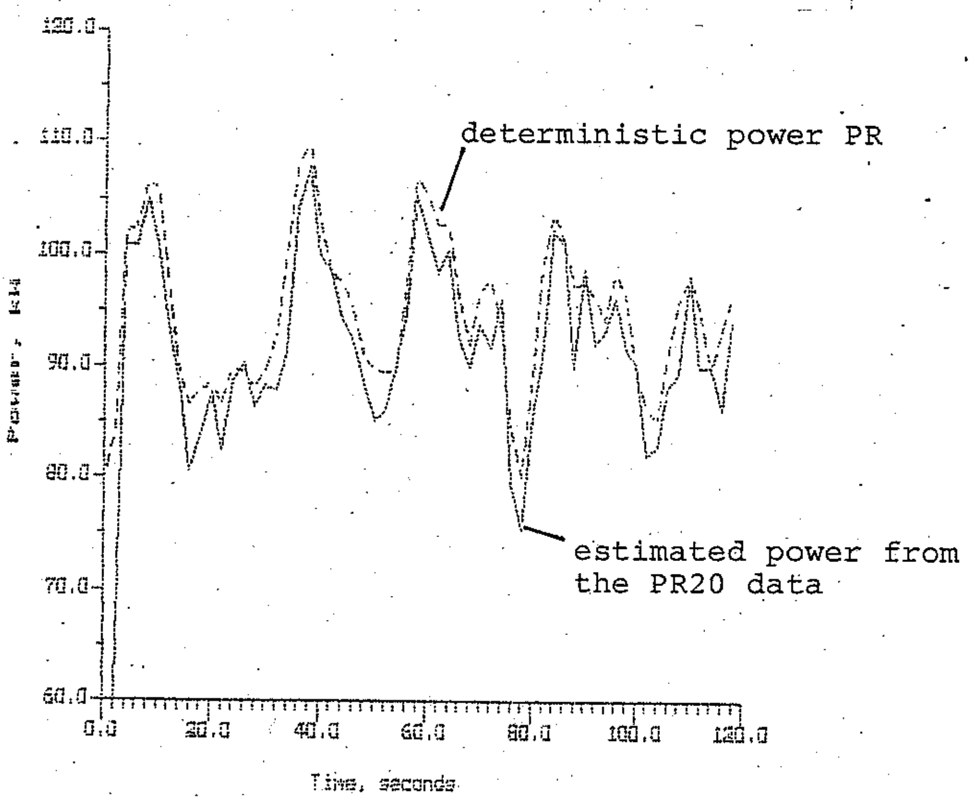

Fig.13. Estimated power PR versus real power. Batch algorithm.

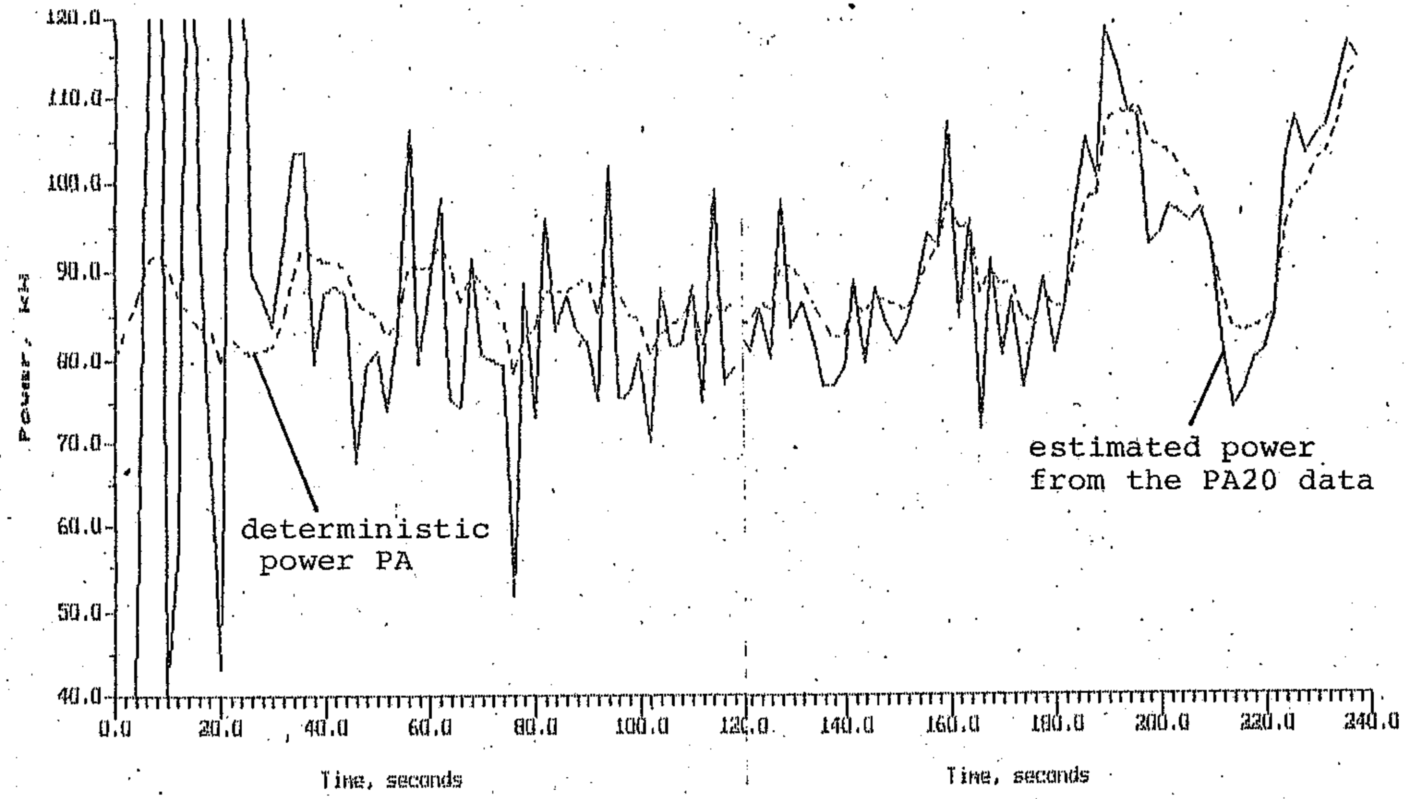

Fig.14. Estimated power PA versus real power.
Recursive algorithm.

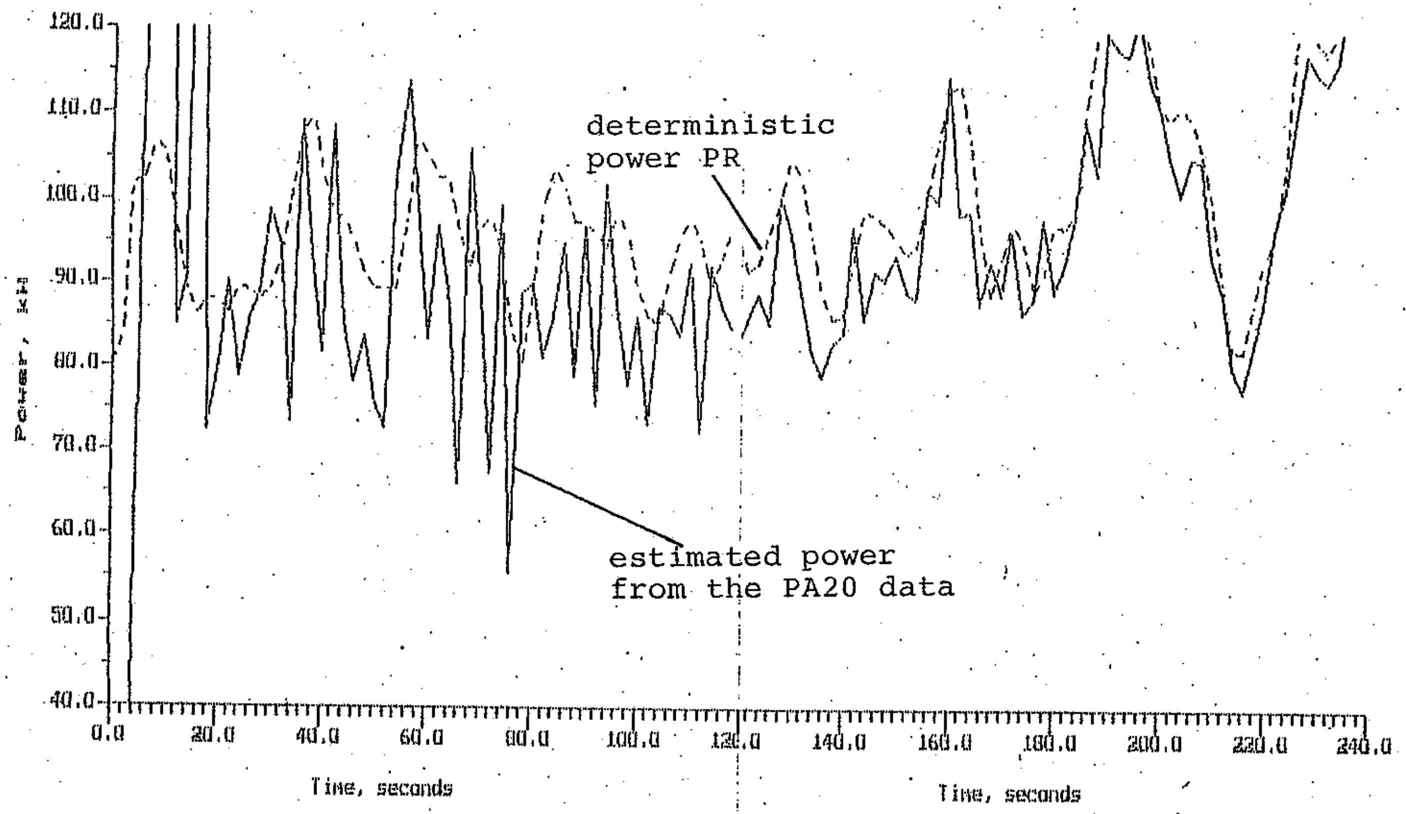

Fig.15. Estimated power PR versus real power. Recursive algorithm.

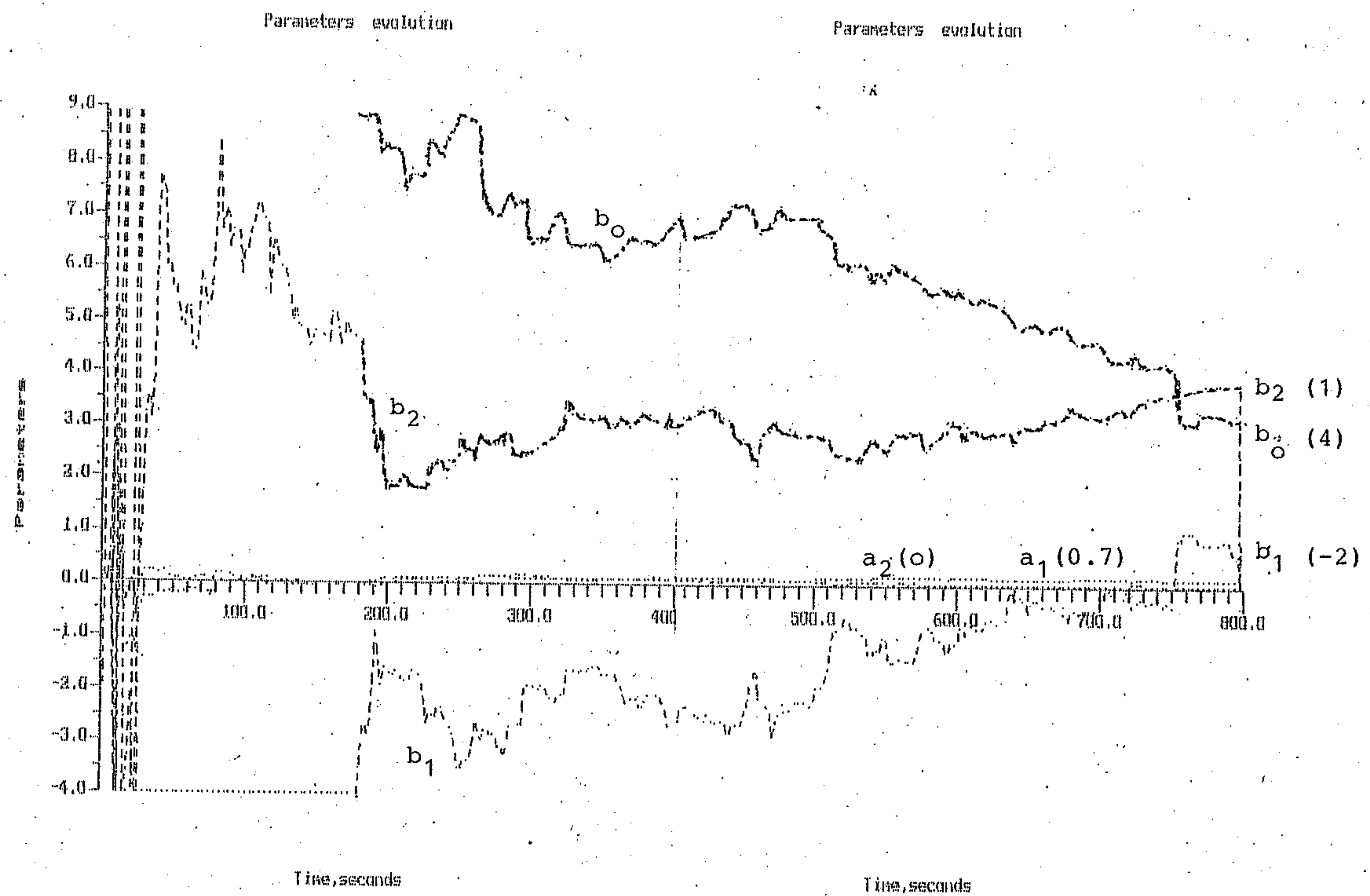

Fig.16. The evolution of the model parameters identification in the recursive algorithm. In parentheses the real values. The processed data PA20.

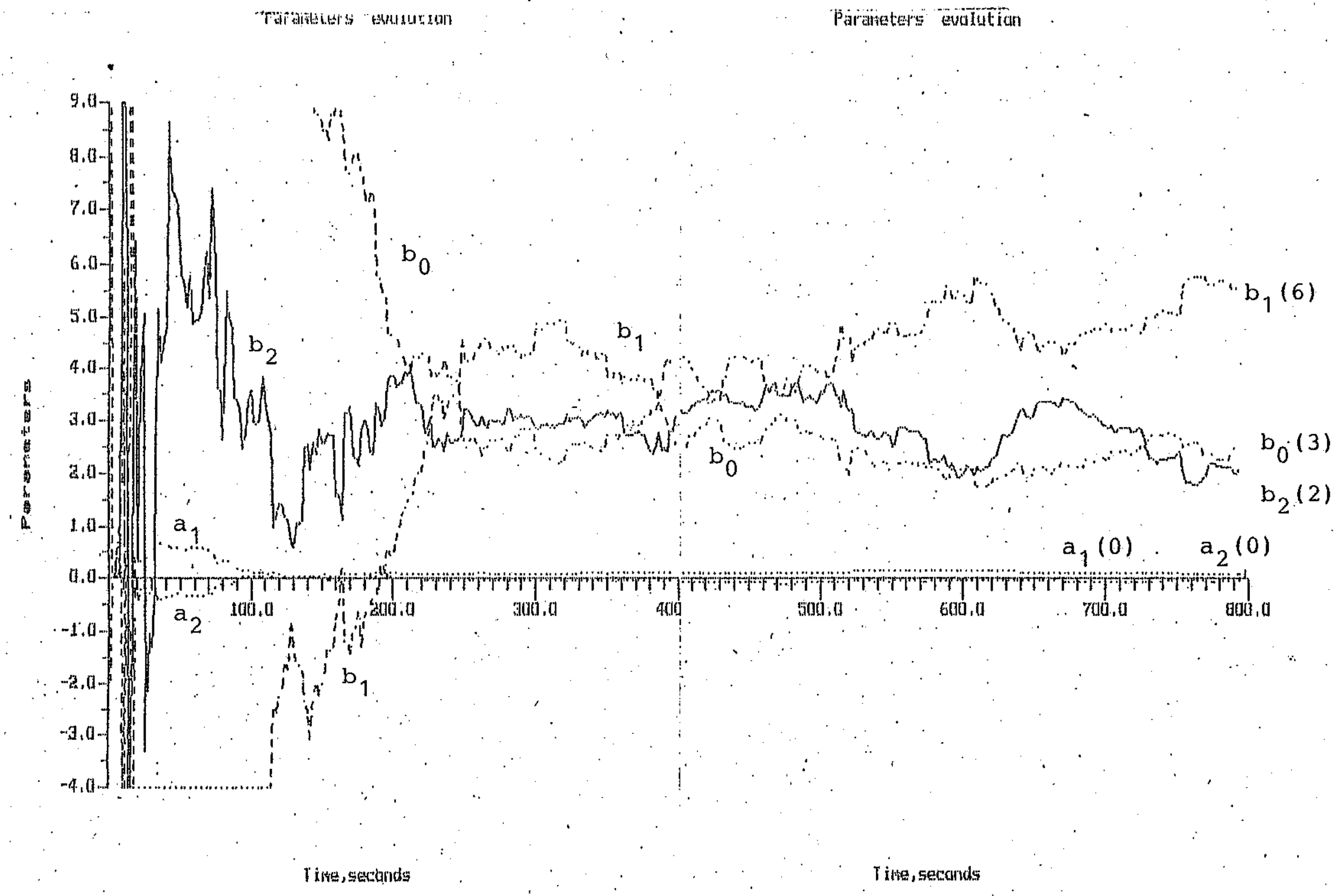

Fig.17. The evolution of the model parameters identification in the recursive algorithm. In parentheses the real values. The processed data: PR20.

4. CONCLUSIONS

-modelling the WT process of power production from wind by experimental identification (using an on-line data processing) is most suitable for control purposes (adaptive control).

-in the process model should be included the wind model as the wind speed acts as a stochastic perturbation. The wind speed model could help, giving adequate prediction, to the optimization of the cut-in/cut-out maneuvers too.

-WT control based on line experimental identification could be used, depending on the WT type and control strategy, with pitch angle , rotor speed or yaw angle regulation.

-for the stall-regulated 500 kW Folkecenter WT an adaptive control could be used for yaw regulation and optimization of the cut-in/cut-out maneuvers.

-the experimental data collected in the operation of the 5oo kW WT will provide the input data for an experimental identification of the process model. This model could be used further in simulating WT limit regimes.

-the incipient calculation programmes for experimental WT identification presented in the final chapter give promising results. The programmes will have further improvements in order to obtain more performing algorithms.

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1. EXPERIMENTAL IDENTIFICATION TECHNIQUES

1.1. Identification based on correlation functions

One calculates the cross correlation function $r_{uy}(k)$ between the input u(t) and the out put y(t) and the autocorrelation function $r_{uu}(t)$ of the output.

Using the Wiener-Hopf equation in discrete time version :

$$r_{uy}(k) = \Sigma h_j * r_{uu}(k-j)$$

the method gives the unit-impulse response h(t); applying further the Laplace transform we obtain the transfer function.

In the frequency domain the Wiener-Hopf equation is

$$H(jw) = Suy(w) / Suu(w)$$

where Suy , cross-spectral density
Suu , power spectral density

1.2. Estimation techniques

This techniques estimate, based on the statistic theory, the parameters of the model which give the best fitted outputs comparing with the real ones. The estimation criteria minimize of a error function using least squares or maximum-likelihood techniques.
For on line estimation one uses recursive algorithms to obtain the input-output model or the state model (Kalman filter) /6,7,8,9/.

The structure of the model should be apriori chosen (based for example on information obtained from a preliminary analytical identification).

The input-output model

The most general structure of a black-box model is (fig.A1):

$$A(q)*y(t) = \frac{B(q)}{F(q)} u(t) + \frac{C(q)}{D(q)} z(t)$$

where

y(t) the output
u(t) the input
z(t) noise
A,B,C,D,E,F polynoms with the backward operator q.

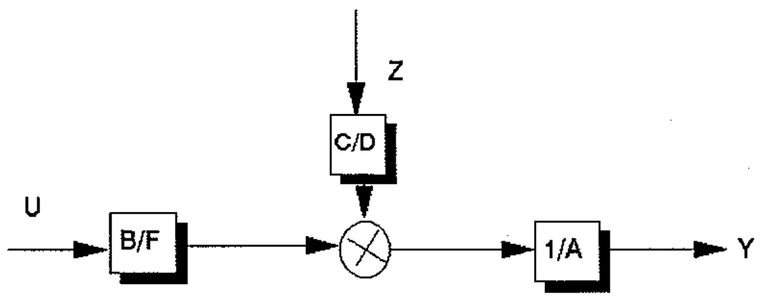

Fig. A1. The general input-output model

The particular models are:

Polynoms	Model name	Symbol
A	Autoregressive	AR
B	Impulse response	FIR
A,B	Autoregressive exogenous	ARX
A,B,C	Autoregr. moving average exogenous	ARMAX
A,C	Autoregressive moving average	ARMA
A,B,D	Repetitive autoreg. exogenous	ARARX
A,B,C,D	Rep. autoreg. moving av. exogenous	ARARMAX
B,F	Output	OE
B,F,C,D	Box Jenkins	BJ

If the model is written in the form $Y = S \theta + Z$
where

Y, output vector
S, previous input-output data matrix
θ, estimated parameters vector
Z, noise vector

the recursive (most used) set of equations for LSQ (Least SQuares) method is:

$$P(k) = \frac{\alpha}{(s^T(k)\ s(k))^{-1}}$$

$$\hat{\theta}(k+1) = \hat{\theta}(k) + M(k+1)[Y(k+1) - s^T(k+1)\ \hat{\theta}(k)]$$
$$M(k+1) = P(k)\ s(k+1)\ [\ \alpha + s\ (k+1)\ P(k)\ s(k+1)]^{-1}$$
$$\alpha = 0.5...1.0$$
$$P(k+1) = [I - M(k+1)\ s^T\ (k+1)]\ P(k)$$

Initial values for θ and P are necessary.

There is also a LSQ method in off line algorithm; The LSQ method is known in ordinary and weighted variants. One of the weighted variants refers to Markov estimate where the weightning matrix R is formed with the inverse of the covariance of the regression equation error.

The INSTRUMENTAL VARIABLES method is a modified LSQ method in which the measured outputs are replaced with determinist values, uncorrelated with the error. A common idea is to use for this purpose the model parameters obtained following an ordinary LSQ.

The Kalman filter

The parameters are regarded as the states of the system and the state equations are used.

One starts from the known state equations:

$$X(k+1) = A\ X(k) + B\ U(k) + v(k)$$
$$Y(k) = C\ X(k) + z(k)$$

The recursive algorithm is:

the estimate state:

$$\hat{X}(k+1) = A\hat{X}(k) + BU(k) + K(k+1)[Y(k+1) - CA\hat{X}(k) - CBU(k)]$$

the gain matrix: $K(k+1) = Q(k+1)C\ \ [CQ(k+1)C^T + Rz\ \ ]^{-1}$

the posteriori covariance $P(k) = cov\ [X(k)]$

the apriori covariance $P(k+1) = [1-K(k+1)C]\ Q(k+1)$

$$Q(k+1) = AP(k)A^T + Rv$$

The process and the filter are shown in the figure A2.

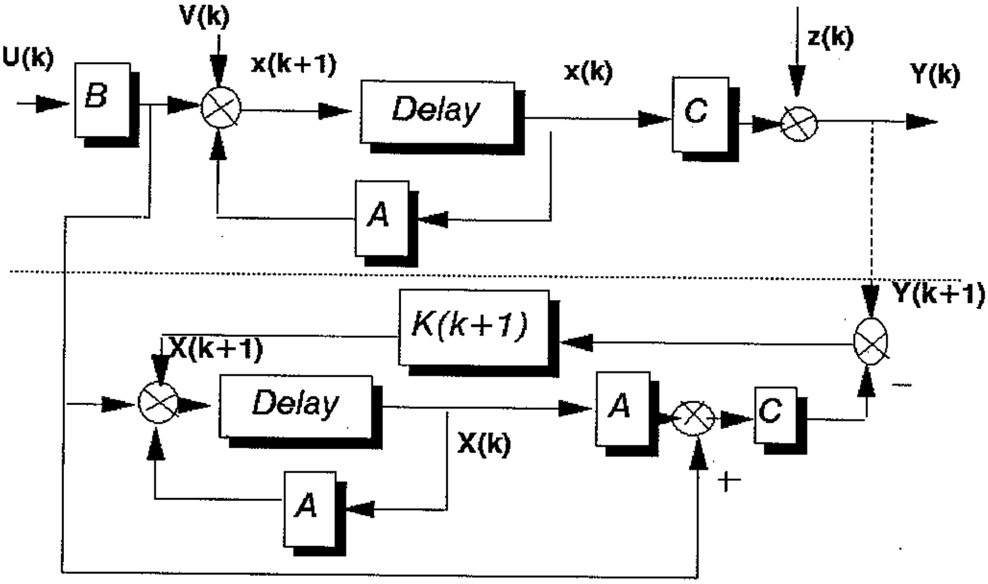

Fig.A2. The process in state terms and the Kalman filter

2.MODELLING THE WIND

The wind speed imposes the stochastic character in modelling a WT process. To consider the wind speed like a white noise perturbation is not realistic so it is necessary to find an appropriate model for the wind too.

The prediction of the wind speed through a proper model serves, except to WT process modelling, to the optimization of start, cut-in, cut-out and stop maneuvers. The maneuvers optimization is particularly useful for WT working with diesel units.

The best strategy is based on wind prediction and on a "hysteresis" effect on the operational diagram of the WT. Through hysteresis we understand different imposed values for the wind speed at the limits of the operational domain depending of the WT condition. For example the cut-in wind speed is fixed at 5.5 m/s and the cut-out speed at 4.7 m/s.

The wind speed prediction is hardly possible od seconds and minutes scale through analytical models. Analytical models could be useful only to simulate sets of wind speed values for tests and validation of the design work /10/. A known technique is to generate sets values with an imposed spectral density S_o (Shinozuka method) /11/:

$$X(t) = \sqrt{2}\ \Sigma A_K \cos\ (w'_K\ t + \phi_K)$$

where $A_K = [S_o(w_K)\ \ \Delta w]$

$\Delta w = (w_u - w_l)/N$, w_u and w_l upper and lower frequency limits

$w_K = w_l + (k-1/2)\Delta w$, $k=1..N$, N-frequencies number

$w'_K = w_K + \delta w$, $\Delta w' << \Delta w$

$\delta w = -\Delta w'/2\ .....\ \Delta w'/2$ stochastic, uniform distribution

$\phi_K = 0...2\pi$ stochastic, uniform distribution

For a spectral density function specific to the wind speed one could choose the Kaimal spectrum:

$$So(f) = \frac{\sigma^2}{2} \frac{fl/\bar{v}}{(1 + 1.5\ fl/\bar{v})^{5/3}}$$

where σ is the variance, l the longitudinal length scale, v the mean wind speed and f the frequency.

Another model is the Vaicatis spectrum

$$So(w) = \frac{2\ K\ F^2}{\pi^2\ [1+(Fw\ /\bar{V}\pi)^2]^{4/3}}$$

where:

K , surface flow coefficient (K = 0.004)

F , turbulence dependent parameter(F = 600 m)

For the on-line control necessities only the experimental identification of the wind speed is possible.

Through the AR or ARMA models the next wind speed values are predicted based od the values measured until the prediction moment.

Usually one applies the Kalman filter for the estimation of the vector 0 parameters, let say for the AR model:

$$V = S\ \Theta + Z.$$

The recursive set of equations is:

$$K(k+1) = Q(k+1)\ S^T(k)\ [S^T(k)*Q(k+1)\ S + Rv]^{-1}$$
$$\hat{\Theta}(k+1) = \hat{\Theta}(k) + K(k+1)\ [V(k+1) - S(k)*\Theta(k)]$$
$$P(k+1) = [1-K(k+1)*S(k)]*Q(k+1)$$
$$Q(k+1) = P(k) + Ro$$

Rv = cov [estimated wind speed values]

$Ro = cov\ [\Theta(k+1)-\Theta(k)]$

As initial values one takes:

Rv = Vtip/10 where Vtip is the common values of measured wind speeds

Ro = 0,01*I

Θ = [0,7 0,3/N.... 0,3/N] and $\Sigma\Theta\ (0) = 1$

N, filter order

With the Kalman filter, several improvements are possible, for example in what concerns the evaluation of the noise covariance Rz; this covariance is taken as the average of the previous predicted errors. Further, the covariance Rz is pondered by increasing the influence of the last residues, considering a weightning parameter:

$$R_{k+1} = (M-1)R_k/M + e_k^2/M$$

A good value for M is around 15 /12/ .

The most simple prediction model is the persistence model. Here the predicted wind speed value is identic with the present measured one. The persistence model is used as reference in comparison with other more elaborated prediction models.

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